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PREFACE.

THE recent introduction of Logarithms into the test required for admission to Addiscombe has induced the Author to publish the following treatise. It will be found useful, he hopes, not only to those who are seeking to enter the Royal Indian College, but also to those who intend to continue, or complete, their mathematical studies.

The body of the work will be sufficient for students who wish to acquire merely the power of *applying* Logarithms to arithmetical operations. The Appendices, at the end, are added for those who are desirous of learning the process of *constructing* Logarithms. For the understanding of the entire work no more previous mathematical knowledge is demanded than that of Arithmetic and the first principles of Algebra.

It is perhaps unnecessary to say there is no pretence of novelty in the proofs: but the Author trusts that he has arranged the subject clearly and methodically, and that he has succeeded in furnishing a serviceable, and a sufficiently copious, set of examples, about the accuracy of which the utmost care has been taken, and to which there are given proper forms according to which the student may work.

The advantage, indeed, of cultivating a neat and connected style of writing out logarithmic calculations cannot be too earnestly insisted on. It will be readily admitted by all who have been engaged either in teaching or in examining. The plan adopted in these pages is one that has been recommended by a practical acquaintance with its usefulness.

Addiscombe, Oct. 1859.

CONTENTS.

APPENDIX.

AN ELEMENTARY TREATISE

ON

LOGARITHMS

CHAPTER I.

GENERAL PROPERTIES OF LOGARITHMS.

1. IF $a^x = m$, $a^y = n$, $a^z = p$, &c. where a is a certain fixed number, and m, n, p, &c. are variable quantities, then,

The corresponding values of x, y, z, &c. are called the logarithms of m, n, p, &c. respectively to the base a.

This may also be expressed thus:—

$$x = \log_a m, \quad y = \log_a n, \quad z = \log_a p, \quad \&c.$$

where m, n, p, &c. are called the natural numbers,
x, y, z, &c. their respective logarithms,
and a the base of the system of logarithms.

It will, of course, necessarily follow that

$$x = \log_a (a^x), \quad y = \log_a (a^y), \quad z = \log_a (a^z).$$

2. Thus, if 3 were the base:

$$\text{since } 3^1 = 3, \quad 3^2 = 9, \quad 3^3 = 27, \quad 3^4 = 81, \quad \&c.$$

we should say that 1, 2, 3, 4, &c. were the respective logarithms of 3, 9, 27, 81, &c. to the base 3; or we might write,

$$1 = \log_3 3, \quad 2 = \log_3 9, \quad 3 = \log_3 27, \quad 4 = \log_3 81, \quad \&c.$$

3. If the logarithms were given as integers, it would be easy enough to find the natural numbers. But it is generally required to find the logarithms when the natural numbers are given; and that would not be always possible by means of the above expressions only.

Thus, from the above equations, where the base is 3, we might at once say what is the value of the natural numbers corresponding to the successive logarithms, 1, 2, 3, &c. from the equations

$$m = 3^1 = 3, \qquad n = 3^2 = 9, \quad \&c.$$

But if the successive natural numbers, 1, 2, 3, &c. were given, it would not be so easy to determine x, y, z, &c. from the equations

$$1 = 3^x, \qquad 2 = 3^y, \quad \&c.$$

And recourse must then be had to the two algebraic formulæ known as the exponential and logarithmic series, of which an account will be given in the Appendices.

4. In any system of logarithms, the logarithm of the base is always equal to 1, and the logarithm of 1 is equal to 0.

For, $\qquad a^1 = a$ and $a^0 = 1$, whatever be the value of a.

Hence $\log_a a = 1$, and $\log_a 1 = 0$.

5. In any system, the logarithm of the *product* of any numbers is equal to the *sum* of the logarithms of those numbers.

$$\text{Let } a^x = m, \quad a^y = n, \quad a^z = p, \ \&c.$$
$$\text{or } \quad x = \log_a m, \quad y = \log_a n, \quad z = \log_a p, \ \&c.$$

$$\begin{aligned}\text{then } m \times n \times p \times \&c. &= a^x \times a^y \times a^z \times \&c.\\ &= a^{x+y+z+\&c.}\end{aligned}$$

or, by the usual definition of a logarithm,

$$\begin{aligned}\log_a \{m \times n \times p \times \&c.\} &= x + y + z + \&c.\\ &= \log_a m + \log_a n + \log_a p + \&c.\end{aligned}$$

6. In any system, the logarithm of the *quotient* of any two numbers is equal to the *difference* of their logarithms.

$$\text{Let} \quad a^x = m, \qquad a^y = n,$$
$$\text{or} \quad x = \log_a m, \qquad y = \log_a n;$$
$$\text{then} \quad \frac{m}{n} = \frac{a^x}{a^y} = a^{x-y},$$

$$\text{or} \log_a\left(\frac{m}{n}\right) = x - y$$
$$= \log_a m - \log_a n.$$

7. In any system, the logarithm of any *power* of a number is equal to the logarithm of that number *multiplied* by the index of that power.

$$\text{Let} \quad a^x = m,$$
$$\text{or} \quad x = \log_a m;$$
$$\text{then} \quad m^t = (a^x)^t$$
$$= a^{tx},$$

$$\text{or} \log_a(m^t) = tx$$
$$= t\log_a m.$$

8. In any system, the logarithm of any *root* of a number is equal to the logarithm of that number *divided* by the index of that root.

$$\text{Let} \quad a^x = m,$$
$$\text{or} \quad x = \log_a m;$$
$$\text{then} \, (m)^{\frac{1}{t}} = (a^x)^{\frac{1}{t}}$$
$$= a^{\frac{x}{t}}$$

$$\text{or} \log_a\left(m^{\frac{1}{t}}\right) = \frac{x}{t}$$
$$= \frac{1}{t}\log_a m.$$

9. It is on these expressions that the use of logarithms is founded; in performing multiplication of numbers by the addition of their logarithms, division by subtraction, involution by multiplication, and evolution by division.

10. Having given the logarithms of numbers to any base, to find the logarithms of the same numbers to any other base.

Let a be the base of the system of given logarithms, b the base of the system of required logarithms, x and y the logarithms of the same number m to these bases respectively.

$$\text{Then } a^x = m = b^y,$$
$$a^{\frac{x}{y}} = b,$$
$$\frac{x}{y} = \log_a b,$$

$$\text{or} \quad y = \frac{1}{\log_a b} x.$$

This quantity $\frac{1}{\log_a b}$ is called the *Modulus* of the system, whose base is b, relative to the system, whose base is a.

11. To prove that $\log_a b \times \log_b a = 1$,

$$\text{let } \log_a b = x, \text{ and } \log_b a = y,$$
$$\text{or} \quad a^x = b, \text{ and } b^y = a;$$
$$\text{hence} \quad a = b^{\frac{1}{x}} = b^y,$$
$$\text{or} \quad \frac{1}{x} = y,$$
$$xy = 1,$$
$$\log_a b \times \log_b a = 1.$$

To prove that $\frac{\log_a m}{\log_b m} = \log_a b$,

$$\textit{let } \log_a m = x, \quad \log_b m = y, \quad \text{and } \log_a b = z,$$

$$\text{or} \qquad a^x = m,\ b^y = m, \text{ and } a^z = b,$$
$$a^x = b^y = a^{yz}$$
$$x = yz$$
$$\frac{x}{y} = z$$

$$\text{or } \frac{\log_a m}{\log_b m} = \log_a b.$$

12. To prove that $\log_a 0 = -\infty$, and that we cannot have the logarithms of negative quantities.

$$a^{-\infty} = \frac{1}{a^{\infty}} = \frac{1}{\infty} = 0,$$
$$\text{hence } \log_a 0 = -\infty.$$

Also a^m cannot be any negative quantity, since a is positive. Therefore we cannot express the logarithm of a negative quantity.

13. Any number *might* be used as a base; but there are only two bases which are really ever used. The one is a number 2·7182818 .... which is denominated e, and is the base of what is called the Napierian or the natural logarithms, the advantage of which, as will be shown in the Appendix, consists in the ease with which logarithms are constructed to this base.

The other base is 10, belonging to the common, or Briggs', system of logarithms, the advantage of which is thus shown.

Let us take any numbers, differing only in the position of the decimal points: —

as 2345, 234·5, 23·45, 2·345;

and

let $\log_{10} 2345$ be known $= 3·370143$;

$$\textit{then} \quad \log_{10} 234·5 = \log_{10} \frac{2345}{10},$$

$$\log_{10} 234{\cdot}5 = \log_{10} 2345 - \log_{10} 10 \text{ (by Art. 6)},$$
$$= 3{\cdot}370143 - 1 \text{ (by Art. 4)},$$
$$= 2{\cdot}370143 ;$$

$$\log_{10} 23{\cdot}45 = \log_{10} \frac{2345}{10^2},$$
$$= \log_{10} 2345 - \log_{10} (10^2),$$
$$= \log_{10} 2345 - 2 \log_{10} 10 \text{ (by Art. 7)},$$
$$= 3{\cdot}370143 - 2$$
$$= 1{\cdot}370143 ;$$

$$\log_{10} 2{\cdot}345 = \log_{10} \frac{2345}{10^3},$$
$$= \log_{10} 2345 - \log_{10} (10^3),$$
$$= 3{\cdot}370143 - 3,$$
$$= {\cdot}370143.$$

and so on.

Hence, in tabulating these logarithms, it is sufficient to put $\log_{10} 2345 = 370143$, without considering the decimal point at all, and this number is then known as the decimal part of the logarithm of all numbers having the figures 2345; the integral part being determined from other considerations, hereafter to be mentioned. (See Arts. **18** and **20.**)

And as this would not be the case in any other system than that whose base is 10, we immediately perceive why, in practice, the common system only is used.

14. The method of computing these logarithms will be given in the Appendix III. At present it may be enough to say that the Napierian logarithms are first calculated; and then the common logarithms are obtained by multiplying by the modulus of the common system, which is represented by

$$\mu = \frac{1}{\log_e 10} = \frac{1}{2{\cdot}302585} = {\cdot}434294. \text{ (See Art. 10.)}$$

15. In the course of this work we shall use only common logarithms; and, therefore, whenever log a occurs, we shall mean $\log_{10}a$, unless the contrary is specified.

16. From the properties proved at Arts. **4—8**, we are often enabled to find the logarithms of many numbers, when we have other logarithms given, without having recourse to the Tables.

Thus: having given log 2 = ·301030, and log 7 = ·845098, to find log 3·5, log 24·5, log 28, log 1960, and log 1·715.

(1)
$$\begin{aligned} 3{\cdot}5 &= \frac{35}{10} = \frac{7}{2} \\ \log\ 3{\cdot}5 &= \log 7 - \log 2 \\ &= {\cdot}845098 - {\cdot}301030 \\ &= {\cdot}544068. \end{aligned}$$

(2)
$$\begin{aligned} 24{\cdot}5 &= \frac{245}{10} = \frac{49}{2} = \frac{7^2}{2} \\ \log\ 24{\cdot}5 &= 2\log 7 - \log 2 \\ &= 1{\cdot}690196 - {\cdot}301030 \\ &= 1{\cdot}389166. \end{aligned}$$

(3)
$$\begin{aligned} 28 &= 4 \times 7 = 2^2 \times 7 \\ \log\ 28 &= 2\log 2 + \log 7 \\ &= {\cdot}602060 + {\cdot}845098 \\ &= 1{\cdot}447158. \end{aligned}$$

(4)
$$\begin{aligned} 1960 &= 7 \times 10 \times 28 = 7^2 \times 2^2 \times 10 \\ \log\ 1960 &= 2\log 7 + 2\log 2 + \log 10 \\ &= 1{\cdot}690196 + {\cdot}602060 + 1 \\ &= 3{\cdot}292256. \end{aligned}$$

(5)
$$\begin{aligned} 1{\cdot}715 &= \frac{1715}{1000} = \frac{343}{200} = \frac{7^3}{2 \times 10^2} \\ \log\ 1{\cdot}715 &= 3\log 7 - \log 2 - 2\log 10 \\ &= 2{\cdot}535294 - {\cdot}301030 - 2 \\ &= {\cdot}234264. \end{aligned}$$

*Examples.**

I.

(1) In a system, whose base is 5, to find the natural number, whose log is 4.

(2) If 81 be a number calculated to two bases, 3 and 9; what is the ratio between the corresponding logarithms?

(3) If log 9 = $\cdot\dot{6}$, what is the base?

(4) Find log 256 to the base $2\sqrt{2}$.

(5) In the common system, find log ·0001.

(6) Given $\sqrt{1000} = 31{\cdot}62278$, find log 31·62278.

(7) Given log 2 = ·301030, and log 9 = ·954242, find log 6, log 405, log 4500, and log 32400.

(8) Given log 8 = ·903090, and log 9 = ·954242, find log 12, and log 13·5.

(9) Given log 2 = ·301030, find log 25, and log 62500.

(10) Given log 2 = ·301030, and log 15 = 1·176091, find log 3, and log 1·8.

(11) Given log 98 = 1·991226, and log 112 = 2·049218, find log. 17·5.

(12) The logarithm of a certain number to the base 3 is n times the logarithm of the same number to the base 2; find n.

(13) The logarithm of a certain number to the base 3 is a; find the logarithm of the same number to the base 5.

(14) The product of two numbers is c, and the logarithm

* *No tables* are to be used in this set of examples.

of the first to the base a is m times the logarithm of the second to the base b; find the numbers.

(15) In a geometrical progression, having given a, s, and r, to find n.

(16) Given $(a^m)^x = b$, to find x.

(17) Given $x^y = y^x$, and $x^m = y^n$, to find x and y.

(18) Given $a^1 \times a^3 \times a^5 \times a^7 \times$ &c. to n terms $= p$, to find n.

CHAP. II.

THE USE OF TABLES OF SIX FIGURES.

17. WE shall begin by pointing out the manner of using tables where the logarithms have six places of decimals; and then we shall show how larger tables may be applied.

We shall take, for the present, the tables published by the Rev. J. Cape, of Addiscombe, as being the most convenient.

18. It will be noticed that only the decimal part of the logarithm is tabulated; for this serves for all natural numbers, which differ only in the position of the decimal point. (See Art. **13.**)

The integral part of a logarithm is called the *characteristic*, or *index;* and may always be determined by the following rule:—

The characteristic of any logarithm is a number one less than the number of integer places in the natural number.

The reason for this rule is, that every number consisting of *one* place of integers (such as 8, or 8·765) lies between 1 and 10; and therefore its logarithm must lie between 0 and 1, or must be 0 together with some decimal.

Again, every number consisting of *two* places of integers (such as 87, or 87·65) lies between 10 and 100; and therefore its logarithm must lie between 1 and 2, that is, must be 1 together with some decimal.

Again, every number consisting of *three* places of integers (such as 876, or 876·5) lies between 100 and 1000; and therefore its logarithm must lie between 2 and 3, or must be 2 together with some decimal.

And so on.

In general, every number consisting of n integers lies

between 10^{n-1} and 10^n; its logarithm, therefore, lies between $n-1$ and n, or is $n-1$ together with some decimal.

19. Hence, referring to a portion of Cape's Logarithms, Table XIX. p. 82, of the second edition, we shall find the following numbers :—

No.	Log.	Prop. Part.
8680	938520	
1	938570	5
2	938620	10
3	938670	15
4	938720	20
5	938770	25
6	938820	30
7	938870	35
8	938920	40
9	938970	45

We have

$$\log 8{\cdot}680 = 0{\cdot}938520$$
$$\log 86{\cdot}81 = 1{\cdot}938570$$
$$\log 868{\cdot}2 = 2{\cdot}938620$$
$$\log 8683 = 3{\cdot}938670$$
$$\log 86840 = 4{\cdot}938720$$

and so on.

20. If, however, the natural number has no integral places, the rule is this : —

The characteristic of the logarithm of a number, which is altogether decimal, is a negative quantity, one more than the number of zeros between the point and the first significant figure.

The reason for this rule is, that a decimal number having *no* such zero (as ·8 or ·8765), lies between ·1 and 1, or $\frac{1}{10}$ and 1, or 10^{-1} and 10^0. Its logarithm, therefore, must lie between -1 and 0, or must be -1 together with some decimal attached.

If there be one zero (as ·08 or ·08765), the number lies

between ·01 and ·1, or $\frac{1}{100}$ and $\frac{1}{10}$, or 10^{-2} and 10^{-1}. It is, therefore, greater than 10^{-2}, and less than 10^{-1}; and the logarithm must lie between -2 and -1, or must be -2 together with some decimal.

If there be *two* zeros (as ·008 or ·008765), the number lies between ·001 and ·01, or $\frac{1}{1000}$ and $\frac{1}{100}$, or 10^{-3} and 10^{-2}. It is, therefore, greater than 10^{-3} and less than 10^{-2}; and the logarithm must lie between -3 and -2, or must be -3 together with some decimal.

And so on.

In general, if there be n zeros, the number lies between $10^{-\overline{n+1}}$ and 10^{-n}; and the logarithm therefore will be $-\overline{n+1}$ together with some decimal.

21. Hence, referring to the table above, we have

$$\begin{aligned}
\log \ \cdot 8685 &= \bar{1}\cdot 938770 \\
\log \ \cdot 08686 &= \bar{2}\cdot 938820 \\
\log \ \cdot 008687 &= \bar{3}\cdot 938870 \\
\log \ \cdot 0008688 &= \bar{4}\cdot 938920 \\
\log \ \cdot 00008689 &= \bar{5}\cdot 938970 \qquad \&c.
\end{aligned}$$

22 The negative sign is placed *over* the characteristic, to denote that it alone is negative; the decimal part, by the principles above, being positive.

23. As the integral part of a logarithm is called the *characteristic* or *index*, so the decimal part is sometimes styled the *mantissa*,—that is, *a handful*, something over and above the characteristic.

Examples.

II.

Find the logarithms of the following numbers :—

(1) 74320 (2) 25·65 (3) 2·421

(4) ·8713 (5) ·002531 (6) ·0000021

Find the natural numbers whose logarithms are respec-*tively the following* quantities:—

(7) 2·621280, (8) 5·763053, (9) $\bar{3}\cdot 797268$.

24. If the natural number have more than four places of significant figures, the additional figures are accounted for by the column of proportional parts; which are computed on the supposition that the differences between the natural numbers are proportional to the differences between the corresponding logarithms.

This proposition, which (as will be shown in the Appendix I. Art. **13**), is approximately true, may be thus stated:—

$$\log(n+\delta)-\log n : \log(n+\delta')-\log n :: \delta : \delta'.$$

25. Thus, referring to the above table on p. 11, we have,

$$\begin{array}{rl} \log 86800 = & 4\cdot938520 \\ \log 86810 = & 4\cdot938570 \\ \hline \log 86810-\log. 86800 = & \qquad\quad 50\,; \end{array}$$

hence if we are required to find the logarithm of any intermediate number, such as 86804, we can say,

$$\log 86804-\log 86800 : \log 86810-\log 86800 :: 4 : 10;$$

and log 86804—log 86800, is called the proportional part for 4.

Therefore, prop. part for 4 : 50 :: 4 : 10;

and similarly for any others.

$$\text{Prop. part for } 1 = \frac{1\times 50}{10} = 5\,;$$

$$\text{,,} \quad 2 = \frac{2\times 50}{10} = 10\,;$$

$$\text{,,} \quad 3 = \frac{3\times 50}{10} = 15\,;$$

$$\text{,,} \quad 4 = \frac{4\times 50}{10} = 20\,; \text{ and so on.}$$

These proportional parts are tabulated, as may be seen, by *being placed* opposite respectively to 1, 2, 3, 4, &c.; and it *is evident*, from the process of finding them, that the la

figure of the proportional part must go under the last figure of the logarithm.

Thus, to find log 86·802:—

$$\begin{array}{lrl}
\text{we have,} & \log 86{\cdot}80 = & 1{\cdot}938520 \\
 & \text{p. p. to } 2 = & \phantom{1{\cdot}9385}10 \\
\hline
\text{hence,} & \log 86{\cdot}802 = & 1{\cdot}938530
\end{array}$$

Examples.

II.

Find the logarithms of the following numbers:

(10) 182·35 (11) ·81346 (12) ·0021734

26. If the natural number consist of six or more figures, the proportional parts for the successive figures may be determined from the same column of proportional parts; observing that, as each successive figure has a local value ten times less than that of the preceding figure, so each succeeding proportional part must be moved off one place to the right hand, as will be seen in the following example.

To find log 23·4783207:

$$\begin{array}{rr|l}
\log 23{\cdot}47 = & 1{\cdot}370513 & \\
\text{p. p. to } 8 = & 148 & \\
\text{,,} \quad 3 = & 5 & 6 \\
\text{,,} \quad 2 = & & 37 \\
\text{,,} \quad 07 = & & \;\;130 \\
\hline
 & 1{\cdot}370666 & 9830;
\end{array}$$

or, log 23·4783207 = 1·370667, neglecting the decimals after the sixth.

It will be observed that we have drawn a vertical line, for the purpose of separating the decimals after the sixth place.

Let it also be borne in mind that, as the proportion at Art. 24 is only approximately true, the proportional parts after the first or second cannot be relied on as being accu-

rately true. We have set them down in the last example, only for the sake of the principle.

Examples.

II.

Find the logarithms of the following numbers:

(13) ·1872109 (14) ·00561345.

27. The converse problem—to find the natural numbers, when the logarithms are given—can readily be seen from the following:

To find the number, whose log is 2·253164.

Given log	=	2·253164
log 1791	=	·253096, the next less in the table.
		68
p. p. to 2	=	48
		200
„ 8	=	194
		60
„ 2	=	48
		&c.

Hence the required number is 179·1282.

Examples.

II.

Find the natural numbers corresponding to the following logarithms:

(15) 1·283746 (16) $\bar{2}$·571835 (17) $\bar{5}$·218710

CHAP. III.

THE USE OF TABLES OF SEVEN FIGURES.

28. THESE tables are, of course, larger than those of six figures; and the mode of their arrangement is somewhat different. The following is a portion of a page taken from Hutton's Logarithms, page 45.

N. 29500. L.469. (45)

N.	O	1	2	3	4	5	6	7	8	9	D	Pro.	
2950	4698220	8367	8515	8662	8809	8956	9103	9251	9398	9545			
51	9692	9839	9986	$\bar{0}134$	0281	0428	0575	0722	0869	1016		147	
52	4701164	1311	1458	1605	1752	1899	2046	2193	2340	2487		1	15
53	2634	2782	2929	3076	3223	3370	3517	3664	3811	3958	147	2	29
54	4105	4252	4399	4546	4693	4840	4987	5134	5281	5428		3	44
55	5575	5722	5869	6016	6163	6310	6457	6604	6750	6897		4	59
56	7044	7191	7338	7485	7632	7779	7926	8073	8219	8366		5	74
57	8513	8660	8807	8954	9101	9248	9394	9541	9688	9835		6	88
58	9982	$\bar{0}129$	0275	0422	0569	0716	0863	1009	1156	1303		7	103
59	4711450	1596	1743	1890	2037	2183	2330	2477	2624	2770		8	118
												9	132

29. In this table we have the natural numbers between 29500 and 29599; and the logarithms are between 4698220 and 4712770. The columns under 1, 2, 3, &c. give only the last four places; the first three being already given in the column under O.

$$\text{Thus } \log 29543 = 4704546$$
$$\log 29578 = 4709688$$

and so on.

It will be seen that at the logarithms of 29513 and 29581, *there is a horizontal* line placed over the 0 in the fourth

place of figures. The significance of this is that the preceding third place of figures here, and all along the same line, is to be increased by 1. And so the form of the table may be preserved.

Thus log 29513 = 4700134, and log 29581 = 4710129.

30. The rules for finding the characteristic are proved at Arts. **18** and **20**, and may be repeated here.

The characteristic of the logarithm is a number one less than the number of integer places in the natural number.

If the natural number have no integer places, then the characteristic is a negative quantity one more than the number of zeros between the decimal point and the first significant figure.

31. Hence, referring to the table given above we shall find:

$$\begin{aligned}
\log\ 295{\cdot}24 &= 2{\cdot}4701752\\
\log\ 29{\cdot}547 &= 1{\cdot}4705134\\
\log\ 2{\cdot}9565 &= 0{\cdot}4707779\\
\log\ {\cdot}29578 &= \bar{1}{\cdot}4709688\\
\log\ {\cdot}029583 &= \bar{2}{\cdot}4710422\\
\log\ {\cdot}0029598 &= \bar{3}{\cdot}4712624
\end{aligned}$$

Examples.

III.

Find the logarithms of the following numbers:

(1) 74320 (2) 25·65 (3) 2·421

(4) ·8713 (5) ·002531 (6) ·0000021

Find the natural numbers corresponding to the following logarithms:

(7) 2·6212802 (8) 5·7630534 (9) $\bar{3}{\cdot}7972675$

Find the logarithms of the following numbers:

(10) 182·35 (11) ·81346 (12) ·0021734

32. If the natural number consist of more than five places, the additional ones can be accounted for from the column of proportional parts, according to the principle stated at Art. **24.**

In these tables it is usual to put, first, a column headed D, which means the difference between two consecutive logarithms. And in the next column, headed *Pro*, are placed the proportional parts corresponding to that difference.

These proportional parts are determined by the principle laid down at Art. **24**; and they are employed according to the following scheme:

33. To find log 23·4783207.

Referring to the tables we find D, the difference between log 23478 and log 23479, to be equal to 185.

$$
\begin{array}{rr|l}
\log 23{\cdot}478 = & 1{\cdot}3706611 & \\
\text{p. p. to } 3 = & 56 & \\
\text{,,} \quad 2 = & 3 & 7 \\
\text{,,} \quad 07 = & & 130 \\
\hline
& 1{\cdot}3706670 & 830
\end{array}
\qquad\qquad D = 185
$$

and therefore, neglecting the places of decimals after the seventh, we have:

$$\log 23{\cdot}478307 = 1{\cdot}3706671.$$

Examples.

III.

Find the logarithms of the following numbers:

(13) ·1872109 (14) ·00561345

Find the natural numbers corresponding to the following logarithms:

(15) 1·2837460 (16) $\bar{2}{\cdot}5718350$ (17) $\bar{5}{\cdot}2187100$

CHAP. IV.

ARITHMETICAL OPERATIONS BY LOGARITHMS.

34. *Multiplication* may be performed by the principle of Art. **5**.

Thus to find the product of 23·7214 and ·01368096:

$$\text{Let } x = 23{\cdot}7214 \times {\cdot}01368096$$
$$\log x = \log 23{\cdot}7214 + \log {\cdot}01368096$$

$$
\begin{array}{rr|l}
\log 23{\cdot}72 = & 1{\cdot}375115 & \\
\text{p. p. to } 1 = & 18 & \\
\text{,, } \quad 4 = & 7 & 3 \\
\log {\cdot}01368 = & \bar{2}{\cdot}136086 & \\
\text{p. p. to } 09 = & 28 & 7 \\
\text{,, } \quad 6 = & 1 & 91 \\
\hline
 & \bar{1}{\cdot}511256 & 91 \\
\log 3245 = & {\cdot}511215 & \\
\cline{2-2}
 & 42 & \\
\text{p. p. to } 3 = & 40 & \\
\cline{2-2}
 & 20 & \\
\text{,, } \quad 1 = & 13 & \\
\end{array}
$$

and therefore $x = {\cdot}324531$.

N.B. It will be observed that we neglect the places to the right of our vertical line; but whenever, as above, the first of these neglected places is more than 5, the place immediately preceding must be increased by 1. Thus, in the above example, since 511256|91 is nearer to 511257 than to 511256, we take care in the subtraction to say *5 from 7 are 2.*

Examples.

IV.

(1) Multiply 2·4671 by ·004386.

(2) Multiply 24000 by ·000783.

(3) Multiply ·5670008 by ·123456.

(4) Multiply together 486·7, 259·4, and 1·2736.

35. *Division* may be performed by the principle of Art. 6.

Thus, to divide 32·7156 by $2{\cdot}6\dot{8}$.

N.B. In this example, when we look out the logarithm of the repeating decimal, we must take the repeating part only so far as to affect the sixth place.

$$
\begin{array}{rcr|ll}
\text{Let} \quad x & = & \multicolumn{3}{l}{32{\cdot}7156 \div 2{\cdot}6\dot{8}} \\
\log x & = & \multicolumn{3}{l}{\log 32{\cdot}7156 - \log 2{\cdot}6\dot{8}} \\
\log 32{\cdot}71 & = & 1{\cdot}514680 & & \\
\text{p. p. to } 5 & = & 66 & & \\
\text{,, } \ 6 & = & 8 & 0 & \\
\cline{3-4}
 & & 1{\cdot}514754 & 0 & \\
\log 2{\cdot}688 & = & 0{\cdot}429429 & & \\
\text{p. p. to } 8 & = & 129 & & \\
\text{,, } \ 8 & = & 12 & 9 & \text{Subtractive.} \\
\text{,, } \ 8 & = & 1 & 29 & \\
\text{,, } \ 8 & = & & 129 & \\
\cline{3-4}
 & & 1{\cdot}085181 & 681 & \\
\log 1216 & = & {\cdot}084934 & & \\
\cline{3-3}
 & & 248 & & \\
\text{p. p. to } 6 & = & 214 & & \\
\cline{3-3}
 & & 340 & & \\
\text{,, } \ 9 & = & 322 & & \\
\cline{3-3}
\end{array}
$$

therefore $x = 12{\cdot}1669$.

36. If the logarithm to be subtracted have a negative characteristic, and in the subtraction something is carried from the mantissa, it must be recollected that this something is positive (see Art. **22**), as will be seen in the following example:

To divide 18·792 by ·000783

$$
\begin{aligned}
\text{Let } x &= 18{\cdot}792 \div {\cdot}000783 \\
\log x &= \log 18{\cdot}792 - \log {\cdot}000783. \\
\log 18{\cdot}79 &= 1{\cdot}273927 \\
\text{p. p. to } 2 &= \phantom{1{\cdot}2739}46 \\
&\ 1{\cdot}273973 \\
\log {\cdot}000783 &= \bar{4}{\cdot}893762 \\
&\ 4{\cdot}380211 \\
\log 24 &= {\cdot}380211
\end{aligned}
$$

Therefore $x = 24000$.

In the subtraction it will be seen that, carrying 1 to the $\bar{4}$, we say 1 and -4 make -3; which, taken from 1, leaves $+4$.

Examples.

IV.

(5) Divide	18·38	by	$2{\cdot}04\dot{2}$
(6) Divide	23·702	by	564·713
(7) Divide	$25{\cdot}\dot{3}\dot{9}$	by	$46{\cdot}\dot{5}0\dot{9}$
(8) Divide	1·32704	by	·0358
(9) Divide	254·90901	by	·00471698
(10) Divide	17385	by	·006988964
(11) Divide	·9649	by	·0275227

37. In a similar way the value of complex fractions may be determined.

Thus, to find $x = \dfrac{24 \times 32{\cdot}156 \times {\cdot}045}{27{\cdot}58 \times {\cdot}176}$

$$\log x = \log 24 + \log 32{\cdot}156 + \log {\cdot}045 - \log 27{\cdot}58 - \log {\cdot}176.$$

In this case, add together all the positive logarithms, and then subtract the sum of all the negative logarithms.

$$\begin{array}{rr}
\log 24 = & 1{\cdot}380211 \\
\log 32{\cdot}15 = & 1{\cdot}507181 \\
\text{p. p. to } 6 = & 81 \\
\log {\cdot}045 = & \bar{2}{\cdot}653213 \\
\hline
 & 1{\cdot}540686 \\
 & {\cdot}686107 \\
\hline
 & {\cdot}854579 \\
\log 7154 = & {\cdot}854549 \\
\hline
 & 30 \\
\text{p. p. to } 5 = & 30
\end{array}
\qquad
\begin{array}{rr}
\log 27{\cdot}58 = & 1{\cdot}440594 \\
\log {\cdot}176 = & \bar{1}{\cdot}245513 \\
\hline
 & {\cdot}686107
\end{array}$$

$$x = 7{\cdot}1545$$

Examples.

IV.

(12) Find the value of $\dfrac{281 \times 2{\cdot}71828}{84000 \times {\cdot}073009}$.

(13) Find the value of $\dfrac{2{\cdot}53 \times {\cdot}00814}{4{\cdot}76509 \times 32{\cdot}14}$.

(14) Find the value of $\dfrac{1185{\cdot}57}{63{\cdot}87009 \times {\cdot}000725}$.

38. ***Involution*** may be performed by the principle of *Art. 7.*

Thus, to find $x = (2{\cdot}17235)^6$:

$$
\begin{array}{rrl}
\log x = & 6 \log 2{\cdot}17235 & \\
\log 2{\cdot}172 = & 0{\cdot}336860 & \\
\text{p. p. to } 3 = & 60 & \\
\text{,, } 5 = & \underline{10} & \underline{0} \\
 & {\cdot}336930 & 0 \\
 & \underline{6} & \\
 & 2{\cdot}021580 & \\
\log 1050 = & \underline{{\cdot}021189} & \\
 & 391 & \\
\text{p. p. to } 9 = & \underline{371} & \\
 & 200 & \\
\text{,, to } 4 = & \underline{165} & \\
 & \&c. &
\end{array}
$$

Therefore $x = 105{\cdot}0948$.

39. If the logarithm has a *negative* characteristic, it must not be forgotten that the number *carried* in the multiplication, from the decimal part, is positive; as will be seen in the following example:—

To find the 4th power of ·0732508:

$$
\begin{array}{rrl}
\text{Let } x = & ({\cdot}0732508)^4 & \\
\log x = & 4 \log {\cdot}0732508 & \\
\log {\cdot}07325 = & \bar{2}{\cdot}864808 & \\
\text{p.p. to } 08 = & \underline{4} & \underline{8} \\
 & \bar{2}{\cdot}864812 & 8 \\
 & & \underline{4} \\
 & \bar{5}{\cdot}459251 & 2 \\
\log 2879 = & \underline{{\cdot}459242} & \\
 & 90 & \\
\text{p.p. to } 06 = & 91 &
\end{array}
$$

Therefore $x = {\cdot}0000287906$.

Examples.

IV.

(15) Find the 7th power of $3\cdot13675$.

(16) Find the 9th power of $1\cdot\dot{2}$.

(17) Find the 14th power of $1\cdot04$.

(18) Find the 4th power of $\cdot07391$.

(19) Find the 5th power of $\cdot0321908$.

(20) Find the 13th power of $\cdot87$.

40. *Evolution* may be performed by the principle of Art. **8.**

Thus, to find $x = \sqrt[5]{25\cdot716}$

$$\begin{array}{rl}
\log x = & \frac{1}{5} \log 25\cdot716 \\
\log 25\cdot71 = & 1\cdot410102 \\
\text{p.p. to } 6 = & 101 \\
 & 5\,)\,\overline{1}\cdot410203 \\
 & \cdot282040 \,|\, 6 \\
\log 1914 = & \cdot281942 \\
 & 99 \\
\text{p.p. to } 4 = & 91 \\
 & 80 \\
\text{p.p. to } 3 = & 68
\end{array}$$

Therefore $x = 1\cdot91443$.

41. If the logarithm has a *negative* characteristic, care must be taken, in the division, to carry to the next place *what is equivalent* to the number borrowed.

$$\text{To find } x = \sqrt[3]{\cdot 00053648}$$

$$\begin{aligned}
\log x &= \tfrac{1}{3}\log \cdot 00053648 \\
\log \cdot 0005364 &= \bar{4}\cdot 729489 \\
\text{p.p. to } 8 &= \phantom{\bar{4}\cdot 7294}65 \\
& \ 3/\overline{\bar{4}\cdot 729554} \\
& \ \bar{2}\cdot 909851 \\
\log 8125 &= \phantom{\bar{2}}\cdot 909823 \\
& \phantom{= \bar{2}\cdot 9098}28 \\
\text{p.p. to } 5 &= \phantom{\bar{2}\cdot 9098}27
\end{aligned}$$

Therefore $x = \cdot 081255$.

Here we say, 3 into $\bar{4}$ $(= -6+2)$ will go $\bar{2}$ and carry 2 (the equivalent), 3 into 27 are 9; and so on as usual.

Examples.

IV.

(21) Find the 6th root of 1234·567.

(22) Find the 8th root of 1·00887.

(23) Find the 50th root of 10.

(24) Find the 5th root of ·0856329.

(25) Find the cube root of ·00052525.

(26) Find the 7th root of ·32705.

(27) Find the cube root of ·0000720509.

(28) Find the cube root of ·22.

(29) Find the 17th root of ·071852.

42. In Involution and Evolution, when *the index is a decimal*, it is better to turn that decimal into a vulgar *fraction*.

IV.

(30) To find ·7th power of 12·3.

$$x = (12{\cdot}3)^{\cdot 7} = (12{\cdot}3)^{\frac{7}{10}}$$
$$\log x = \tfrac{7}{10} \log 12{\cdot}3$$

(31) To find ·7th root of 12·3.

$$x = (12{\cdot}3)^{\frac{1}{\cdot 7}} = (12{\cdot}3)^{\frac{10}{7}}$$
$$\log x = \tfrac{10}{7} \log 12{\cdot}3$$

(32) Raise $\cdot\dot{2}$ to the power of $\dot{2}$.

(33) Extract the ·5th root of ·5.

43. Proportion may be worked out by means of the foregoing principles.

IV.

(34) To find a fourth proportional to 24, 13·76, and ·05.

Let x be the fourth proportional required;

then $24 : 13{\cdot}76 :: {\cdot}05 : x$

$$x = \frac{13{\cdot}76 \times {\cdot}05}{24}$$
$$\log x = \log 13{\cdot}76 + \log {\cdot}05 - \log 24.$$

(35) To find a third continued proportional to 1·7 and ·35.

Let x be the number required;

then $1{\cdot}7 : {\cdot}35 :: {\cdot}35 : x$

$$x = \frac{({\cdot}35)^2}{1{\cdot}7}$$
$$\log x = 2 \log {\cdot}35 - \log 1{\cdot}7.$$

(36) To find a mean proportional between 2543 and ·1726.

$$2543 : x :: x : {\cdot}1726$$
$$x^2 = 2543 \times {\cdot}1726$$
$$2 \log x = \log 2543 + \log {\cdot}1726.$$

CHAP. V.

MISCELLANEOUS EXAMPLES.

44. In finding the value of a complicated expression, it will be convenient to write down carefully the equation of log x, collect the positive quantities together, and also the negative, and subtract the latter from the former.

$$\text{To find } x = \frac{\sqrt{\cdot 00426} \times (\cdot 357)^4}{(\cdot 0468)^3 \times \sqrt[4]{\cdot 579}} \times 234\cdot 567$$

$$\log x = \frac{\log \cdot 00426}{2} + 4 \log \cdot 357 - 3 \log \cdot 0468$$

$$- \frac{\log \cdot 579}{4} + \log 234\cdot 567.$$

$$\begin{array}{lcl}
\dfrac{\log \cdot 00426}{2} = \dfrac{\bar{3}\cdot 629410}{2} & = & \bar{2}\cdot 814705 \\
4 \log \cdot 357 = 4 \times \bar{1}\cdot 552668 & = & \bar{2}\cdot 210672 \\
\log 234\cdot 5 & = & 2\cdot 370143 \\
\text{p. p. to } 6 & = & 111 \\
\quad ,, \quad 7 & = & 13|0 \\
\hline
 & & \bar{1}\cdot 395644
\end{array}$$

$$\begin{array}{lcl}
3 \log \cdot 0468 = 3 \times \bar{2}\cdot 670246 & = & \bar{4}\cdot 010738 \\
\dfrac{\log \cdot 579}{4} = \dfrac{\bar{1}\cdot 762679}{4} & = & \bar{1}\cdot 940670 \\
\hline
 & & \bar{5}\cdot 951408
\end{array}$$

$$\begin{array}{rr}
 & \bar{1}\cdot 395644 \\
 & \bar{5}\cdot 951408 \\
\hline
\log x = & 3\cdot 444236 \\
\log 2781 = & \cdot 444201 \\
\hline
 & 35 \\
\text{p. p. to } 2 = & 31 \\
\hline
 & 40 \\
,, \quad 3 = & 47 \\
\hline
\end{array}$$

Therefore, $x = 2781\cdot 23$

Examples.

V.

(1) Find the value of $\sqrt[5]{\dfrac{57{\cdot}06 \times 3250{\cdot}01}{1000 \times 467}}$.

(2) Find the value of $\dfrac{\sqrt[5]{2{\cdot}43} \times \sqrt[3]{1{\cdot}002}}{({\cdot}0024)^{\frac{1}{7}}} \times (2400)^{\frac{1}{9}}$.

(3) Find the value of $\dfrac{(\frac{13}{14})^{\frac{1}{3}} \times ({\cdot}004)^{\frac{1}{2}} \times 8{\cdot}46}{15 \times ({\cdot}049)^2 \div ({\cdot}186)^{\frac{1}{9}}}$.

(4) Find the value of $\left\{\dfrac{{\cdot}00342 \times (10{\cdot}5)^{\frac{2}{5}}}{\frac{29}{15} \times (\frac{{\cdot}056}{3{\cdot}7})^{\frac{1}{3}}}\right\}^{\frac{1}{4}}$.

(5) Find the value of $\left\{\frac{(23)^2}{25} \times ({\cdot}024)^{\frac{1}{7}}\right\}^{\frac{1}{9}}$.

(6) Find the value of $({\cdot}005234)^{\frac{2}{7}} \div (\frac{24}{{\cdot}017})^{\frac{1}{3}}$.

(7) Find a third proportional to $\sqrt[3]{{\cdot}5}$ and $7\sqrt{({\cdot}04)^7}$.

(8) Find a mean proportional between $\dfrac{\sqrt[7]{15{\cdot}92}}{{\cdot}00526}$ and $\dfrac{\sqrt[3]{{\cdot}0182}}{(196)^5}$.

(9) Find a fourth proportional to $\sqrt[3]{{\cdot}31}$, $({\cdot}41)^2$, and $\sqrt[5]{{\cdot}054321}$.

(10) Find a fourth proportional to the fourth roots of ·27, ·16, and ·12.

(11) Find a mean proportional between $\sqrt[7]{3564}$ and $\sqrt[3]{25}$.

(12) Find a mean proportional between $\sqrt[3]{3567}$ and $\frac{1}{5}\sqrt{{\cdot}004595}$.

(13) Find a third proportional to the cube roots of 1·05 and ·097.

(14) Find a third proportional to 1·3 and $\dfrac{\sqrt[3]{43}}{822{\cdot}108}$.

(15) Find a fourth proportional to $\sqrt[3]{{\cdot}00058309}$, $({\cdot}2839)^3$, and $\sqrt[7]{\dfrac{{\cdot}018}{25}}$.

(16) Find a fourth proportional to $({\cdot}00234)^{\frac{2}{3}}$, $(5{\cdot}00234)^{\frac{3}{5}}$, and $(982{\cdot}5)^{\frac{1}{2}}$.

(17) Find a mean proportional between $\sqrt[5]{{\cdot}01}$ and $({\cdot}\dot{2}\dot{0})^4$.

(18) Find a mean proportional between $\sqrt[3]{{\cdot}03}$ and $({\cdot}000529807)^5$.

(19) Find a third proportional to $({\cdot}948)^5$ and $({\cdot}00052653)^{\frac{1}{3}}$.

(20) Find a mean proportional between $\sqrt[3]{387{\cdot}908}$ and $({\cdot}0187)^{\frac{2}{3}}$.

(21) Find the difference between the series $1+\frac{20}{21}+(\frac{20}{21})^2$ + &c. to infinity, and the same series to 100 terms.

(22) Find the difference between the series $1+\frac{4}{25}+(\frac{4}{25})^2$ + &c. to infinity, and the same series to 20 terms.

(23) Find the value of

$$\frac{{\cdot}2\times{\cdot}4\times{\cdot}8\times1{\cdot}6 \text{ \&c. to 15 terms}}{{\cdot}5\times2{\cdot}5\times12{\cdot}5\times62{\cdot}5 \text{ \&c. to 10 terms}}.$$

(24) Find the value of x in the equation,

$$\sqrt[3]{(1-x)(1-x+x^2)(1+x)(1+x+x^2)(1-x^6)^{\frac{2}{3}}}=\sqrt[6]{({\cdot}4)^5}$$

45. If the terms of the expression be connected by + and −, some device must be adopted, in order to render the *application of logarithms* serviceable.

Thus, to find the value of $\dfrac{(1{\cdot}27)^{13}-2}{(1{\cdot}27)^{13}+2}$.

$$\text{Let } x = \frac{(1{\cdot}27)^{13}-2}{(1{\cdot}27)^{13}+2} = \frac{y-2}{y+2}$$

$$\text{where } y = (1{\cdot}27)^{13}$$

$$\log y = 13 \log 1{\cdot}27$$

$$\text{from which } y = 22{\cdot}359$$

$$x = \frac{y-2}{y+2} = \frac{20{\cdot}359}{24{\cdot}359}$$

$$\log x = \log 20{\cdot}359 - \log 24{\cdot}359$$

$$\text{from which } x = {\cdot}83579.$$

(25) Find the value of $\dfrac{(1{\cdot}0975)^{13}-(1{\cdot}015)^{13}}{(24871{\cdot}53)^{\frac{1}{5}}}$.

(26) Find the value of $\dfrac{(4{\cdot}31)^{\frac{5}{2}} - ({\cdot}018)^{\frac{2}{5}}}{({\cdot}095)^{3}}$.

(27) Find the value of $\dfrac{(1{\cdot}034)^{25}-1}{(1{\cdot}034)^{25}+1}$.

CHAP. VI.

EXPONENTIAL EQUATIONS.

46. Exponential equations are those where the unknown quantity appears in the index or exponent of another quantity. The method of applying logarithms, so as to obtain the unknown quantity, will be seen from the following example:

$$3^{3x} \times 7^{2x-5} = 5^{x-3} \times 11^{x} \times 13^{4-x}$$

$$3x \log 3 + (2x-5)\log 7 = (x-3)\log 5 + x \log 11 + (4-x)\log 13$$

$$x = \frac{5 \log 7 + 4 \log 13 - 3 \log 5}{3 \log 3 + 2 \log 7 + \log 13 - \log 5 - \log 11}$$

$$= \frac{4{\cdot}225490 + 4{\cdot}455772 - 2{\cdot}096910}{1{\cdot}431363 + 1{\cdot}690196 + 1{\cdot}113943 - {\cdot}698970 - 1{\cdot}041393}$$

$$= \frac{8{\cdot}681262 - 2{\cdot}096910}{4{\cdot}235502 - 1{\cdot}740363}$$

$$= \frac{6{\cdot}584352}{2{\cdot}495139}$$

$$\log x = \log 6{\cdot}584352 - \log 2{\cdot}495139$$

$$\begin{array}{rr|l}
\log 6{\cdot}584 = & 0{\cdot}818490 & \\
\text{p. p. to } 3 = & 20 & \\
\text{,,} \quad 5 = & 3 & 3 \\
\text{,,} \quad 2 = & & 13 \\
\hline
 & {\cdot}818513 & 43 \\
\log 2{\cdot}495 = & {\cdot}397070 & \\
\text{p. p. to } 1 = & 17 & \\
\text{,,} \quad 3 = & 5 & 3 \\
\text{,,} \quad 9 = & 1 & 57 \\
\hline
 & {\cdot}421419 & 56 \\
\log\ 2638 = & {\cdot}421275 & \\
\hline
 & 145 & \\
\text{p. p. to } 9 = & 148 &
\end{array}$$

And therefore $x = 2{\cdot}6389$

Examples.

VI.

(1) Solve the equation $2^{3x} \times 7^{4x-1} = 13^{5-x} \times 17^{2x-1} \times 19^x$.

(2) Find x from the proportion $5^{2x} : 7^{3x} :: 13^{5-2x} : 19^{4-x}$.

(3) Solve the equation $(a^2-b^2)^{x-1} \times (a+b)^x = (a-b)^{2x}$.

(4) Find x from the equation $\left(3^{\frac{1}{3}} \times 5^{-\frac{1}{4}}\right)^{2x} = 2^{\frac{1}{2}} \times 7^{-\frac{1}{3}}$.

(5) Find x from the equation $c^x + c^{-x} = 4m$.

(6) Find x and y from the equations $\left.\begin{aligned} 2^x \times 3^y &= 560 \\ 5x &= 7y \end{aligned}\right\}$

(7) Find x and y from the equations $\left.\begin{aligned} 9^x \times 8^y &= 864 \\ 10x &= 9y \end{aligned}\right\}$

(8) Find x and y from the equations $\left.\begin{aligned} 3^{3x} &= 5^{3y+4} \\ 5y &= 2x \end{aligned}\right\}$

(9) Find x and y from the equations $\left.\begin{aligned} 3^{x+y} \times 2^{-x} &= 20 \\ 2x &= 5y \end{aligned}\right\}$

(10) Find x and y from the equations $\left.\begin{aligned} 14^y &= 91x \\ 5^{y+2} &= 7{\cdot}6x^2 \end{aligned}\right\}$

(11) Find x and y from the equations $\left.\begin{aligned} x^y &= y^x \\ x^3 &= y^2 \end{aligned}\right\}$

(12) Solve the equation $3^{x^2-4x+5} = 1200$.

(13) If $2^1 \times 2^3 \times 2^5 \times 2^7 \times$ &c. to n terms $= 33554432$, find the value of n.

(14) How many times must 33·04 be multiplied by 8, that it may be equal to 1419·06 multiplied the same number of times by 5?

(15) In the geometrical progression, 1, 3, 9, 27, &c., find *how many terms* will together amount to 121.

(16) How many terms of the series $\frac{1}{3}+\frac{1}{2}+\frac{3}{4}+\frac{9}{8}+$&c. will be required to make up $\frac{2059}{192}$?

(17) How many terms of the series, $\frac{16}{27}-\frac{8}{9}+\frac{4}{3}-2+$&c. will amount to $4\frac{31}{108}$?

N.B. In this example we shall arrive at the expression $(-\frac{3}{2})^n=-\frac{2187}{128}$. And knowing that we cannot take the logarithm of the negative quantity $-\frac{3}{2}$ (see Art. **12**), we must alter the above expression. For since $(-\frac{3}{2})^n$ is negative, n must be odd; and $\therefore$ $(-\frac{3}{2})^n=-(\frac{3}{2})^n$. Whence $(\frac{3}{2})^n=\frac{2187}{128}$. If, on the other hand, $(-r)^n$ were positive, as in the next example, n must be even, and $(-r)^n=+r^n$.

(18) How many terms of the series $\frac{1}{5}-\frac{2}{15}+\frac{4}{45}-$&c. will amount to $\frac{133}{1215}$?

CHAP. VII.

COMPOUND INTEREST.

47. By the ordinary rules of Arithmetic the calculation of compound interest is often very tedious, and in some cases impossible: but, by the employment of logarithms, the computation is much simplified.

Let P be the principal (expressed in pounds sterling), which is put out at compound interest, at the rate of r£ for every £1 per annum; that is, at $100r$ per cent. per annum, and in n years let the sum amount to A£.

In 1st year £1 amounts to - $(1+r)$£
therefore P£ will amount to $P(1+r)$£

In 2nd year the amount of $(1+r)$ is $(1+r)+r(1+r)$
or $(1+r)^2$
and P will amount to $P(1+r)^2$

In 3rd year the amount of $(1+r)^2$ is $(1+r)^2+r(1+r)^2$
or $(1+r)^3$
and P will amount to $P(1+r)^3$
and so on.

Therefore, in n years P will amount to $P(1+r)^n$; and hence $A=P(1+r)^n$ is the general formula for compound interest.

In this formula if any three of the quantities A, P, r, or n are given, we can find the 4th.

48. *What is the amount of £250 in 10 years at 4 per cent. per annum?*

$$\text{Here } P = 250,\ n = 10,\ r = \frac{4}{100} = \cdot 04$$

$$A = 250\,(1 + \cdot 04)^{10}$$
$$= 250\,(1\cdot 04)^{10}$$
$$\log A = \log 250 + 10 \log 1\cdot 04$$

$$\begin{array}{rr}
\log\ 1\cdot 04 = & \cdot 017033 \\
 & 10 \\
\hline
 & \cdot 170330 \\
\log\ 250 = & 2\cdot 397940 \\
\hline
 & 2\cdot 568270 \\
\text{but } \log 3700 = & \cdot 568202 \\
\hline
 & 68 \\
\text{p. p. to } 6 = & 70 \\
\end{array}$$

$A = 370\cdot 06£ = £370$ 1*s.* $2\frac{3}{8}$*d.* Answer.

N.B. If the question had been to find the *compound interest* on the above, we should subtract £250 from the last result, and say the answer was £120 1*s.* $2\frac{3}{8}$*d.*

49. Find what sum, put out to compound interest, at 4 per cent. per annum, will amount to £370 1*s.* $2\frac{3}{8}$*d.* in 10 years?

$$\text{Here } A = 370\cdot 06,\ n = 10,\ r = \tfrac{4}{100} = \cdot 04$$

$$P = \frac{A}{(1+r)^n} = \frac{370\cdot 06}{(1\cdot 04)^{10}}$$

$$\log P = \log 370\cdot 06 - 10 \log 1\cdot 04$$

$$\begin{array}{rr}
\log 370\cdot 0 = & 2\cdot 568202 \\
\text{p.p. to } 6 = & 70 \\
\hline
 & 2\cdot 568272 \\
10 \log 1\cdot 04 = 10 \times \cdot 017033 = & \cdot 170330 \\
\hline
 & 2\cdot 397942 \\
\log 250 = & \cdot 397940 \\
\hline
\end{array}$$

Therefore $P = £250$. Answer.

50. In what time will £250 amount to £370 1s. $2\frac{3}{8}$d., compound interest being reckoned at the rate of 4 per cent. *per annum?*

$$\text{Here } P = 250, \; A = 370{\cdot}06, \; r = {\cdot}04$$
$$A = P(1+r)^n$$
$$370{\cdot}06 = 250\,(1{\cdot}04)^n$$
$$n = \frac{\log 370{\cdot}06 - \log 250}{\log 1{\cdot}04}$$
$$n = \frac{2{\cdot}568272 - 2{\cdot}397940}{{\cdot}017033}$$
$$= \frac{{\cdot}170332}{{\cdot}017033}$$
$$= 10 \text{ years. Answer.}$$

51. At what rate per cent. per annum will £250 amount to £370 1*s*. $2\frac{2}{5}$*d*., at compound interest, in 10 years?

$$\text{Here } A = 370{\cdot}06, \; P = 250, \; n = 10$$
$$A = P(1+r)^n$$
$$(1+r)^n = \frac{A}{P}$$
$$\log(1+r) = \frac{\log A - \log P}{n}$$
$$= \frac{{\cdot}170332}{10}$$
$$= {\cdot}017033$$
$$\log 1{\cdot}04 = {\cdot}017033$$
$$\text{Therefore} \quad 1 + r = 1{\cdot}04$$
$$r = {\cdot}04$$

Rate per cent. $= 100r = 4$. Answer.

52. If the interest be payable every m^{th} part of a year, it is evident that we must make £1 gain $\frac{r}{m}$ £ for each time payment is made; and that there will be mn such times; and hence

$$A = P\left(1 + \frac{r}{m}\right)^{mn}$$

53. Find the amount of £250 in 10 years, at the rate of 4 per cent. per annum, compound interest being payable every 2 months.

Here $P = 250$, $n = 10$, $m = 6$

$m\,n = 60$, the number of payments.

$\frac{r}{m} = \frac{4}{600} = \frac{1}{150}$, the rate on £1 for every 2 months.

$$A = P\left(1 + \frac{1}{150}\right)^{60}$$
$$= 250\left(\frac{151}{150}\right)^{60}$$

$\log A = \log 250 + 60\,(\log 151 - \log 150)$.

N.B. The fraction $\frac{151}{150}$ is retained, in order to avoid recurring decimals, into which that fraction might be reduced.

$$\begin{array}{rr}
\log 151 = & 2{\cdot}178977 \\
\log 150 = & 2{\cdot}176091 \\
\hline
 & {\cdot}002886 \\
 & 60 \\
\hline
 & {\cdot}173160 \\
\log 250 = & 2{\cdot}397940 \\
\hline
 & 2{\cdot}571100 \\
\log 3724 = & {\cdot}571010 \\
\hline
 & 90 \\
\text{p. p. to } 8 = & 93 \\
\hline
\end{array}$$

Therefore, $A = £372{\cdot}48 = £372\ 9s.\ 7\frac{1}{2}d.$ Answer.

54. Find what sum will amount to £372 9*s*. 7½*d*. in 10 years, at the rate of 4 per cent. per annum, compound interest being payable every 2 months.

Here $A = 372{\cdot}48$, $mn = 60$, $\frac{r}{m} = \frac{1}{150}$,

$$372{\cdot}48 = P\left(1 + \frac{1}{150}\right)^{60}$$
$$P = 372{\cdot}48\left(\frac{150}{151}\right)^{60}$$
$$\log P = \log 372{\cdot}48 + 60\,(\log 150 - \log 151)$$

$$\begin{array}{rr} \log 150 = & 2\cdot176091 \\ \log 151 = & 2\cdot178977 \\ \hline & \bar{1}\cdot997114 \\ & 60 \\ \hline & \bar{1}\cdot826840 \\ \log 372\cdot4 = & 2\cdot571010 \\ \text{p.p. to } 8 = & 93 \\ \hline & 2\cdot397943 \\ \log 250 = & \cdot397940 \\ \hline \end{array}$$

Therefore $P = £250$. Answer.

55. In how many years will £250 amount to £372 9*s.* 7⅕*d.*, compound interest being reckoned at the rate of 4 per cent. per annum, and being payable every 2 months?

$$\begin{aligned} \text{Here } 372\cdot48 &= 250\left(\tfrac{151}{150}\right)^{6n} \\ 6n\{\log (151-\log 150\} &= \log 372\cdot48 - \log 250 \\ 6n \times \cdot002886 &= \cdot173160 \\ 6n &= \frac{\cdot173160}{\cdot002886} \\ &= 60 \\ n &= 10 \text{ years. Answer.} \end{aligned}$$

56. At what rate per cent. per annum will £250 amount to £372 9*s.* 7⅕*d.* in 10 years, compound interest being payable every 2 months?

$$\begin{aligned} \text{Here } 372\cdot48 &= 250\left(1+\frac{r}{6}\right)^{60} \\ 60 \log\left(1+\frac{r}{6}\right) &= \log 372\cdot48 - \log 250 \\ \log 372\cdot48 &= 2\cdot571100 \\ \log 250 &= 2\cdot397940 \\ \hline &\quad \cdot173160 \end{aligned}$$

$$\log\left(1+\frac{r}{6}\right) = \cdot 002886$$

$$\log 1\cdot 006 \quad = \underline{\cdot 002598}$$

$$288$$

$$\text{p.p. to } 6 \quad \doteq \quad \underline{259}$$

$$290$$

&c.

$$1+\frac{r}{6} = 1\cdot 00\dot{6}$$

$$\frac{r}{6} = \cdot 00\dot{6} = \tfrac{6}{900} = \tfrac{1}{150}$$

$$r = \tfrac{6}{150} = \tfrac{1}{25}$$

$100r$=rate per cent. per annum=4. Answer.

Examples.

VII.

(1) What is the amount of £1000, put out to compound interest, at the rate of $3\frac{1}{2}$ per cent. per annum, in 50 years?

(2) Find the compound interest on £1734 17*s.* 6*d.* for 12 years, at $2\frac{1}{4}$ per cent. per annum.

(3) What sum, put out to compound interest, at $3\frac{1}{4}$ per cent. per annum, will amount to £1000 in 25 years?

(4) Find in what time, at compound interest, reckoning 5 per cent. per annum, will £100 amount to £1000.

(5) The compound interest of £534 10*s.* for 5 years is £131 11*s.* 8*d.* Find the rate per cent.

(6) At what rate will £300 amount to £500 in 4 years at compound interest?

(7) What is the amount of £2639 16*s.* $3\frac{1}{2}$*d.* in five years at 4 *per cent.* per annum, compound interest being payable *monthly?*

(8) Find the compound interest on £580 15*s.* in 6 years at 3 per cent., the interest being payable every half year.

(9) Find the amount of £750 10*s.* in 20 years, at $3\frac{1}{2}$ per cent. per annum compound interest, payable quarterly.

(10) What sum will amount to £839 7*s.* $1\frac{1}{2}$*d.* in 15 years, at $3\frac{1}{2}$ per cent. per annum, compound interest being paid every quarter?

(11) What is the compound interest, payable every 9 months, on a sum of money which has amounted to £8476 10*s.* 6*d.* in 27 years, the interest being reckoned at the rate of 5 per cent. per annum?

(12) In how many years would £10 amount to £1000 if put out at compound interest at 4 per cent. per annum, payable monthly?

(13) In how many years will £2653 7*s.* 6*d.*, invested at $3\frac{1}{2}$ per cent. per annum, compound interest, payable quarterly, amount to £3327 18*s.* 1·104*d.*?

(14) At what rate per cent. will £120 13*s.* 4*d.* gain £50 at compound interest for 8 years, interest being paid every quarter?

(15) If £660 when put out to interest, payable quarterly, amounts to £889 4*s.* in 6 years, what is the rate per cent.?

(16) At what rate per cent. per annum will £1000 amount to £1500 in 12 years, compound interest being payable every half year?

(17) A person puts out £20 at 5 per cent. per annum, compound interest, and at the end of every succeeding year puts out an equal sum on the same terms. Find the amount at the end of 20 years.

N.B. We shall have in this example,

$$\begin{aligned}\text{The whole sum} &= 20\,(1{\cdot}05)^{20} + 20\,(1{\cdot}05)^{19} + \dots 20\,(1{\cdot}05)\\ &= 20\,(1{\cdot}05)\left\{\frac{(1{\cdot}05)^{20}-1}{1{\cdot}05-1}\right\}\\ &= 420\,\{(1{\cdot}05)^{20}-1\}.\end{aligned}$$

APPENDIX.

I.

THE LOGARITHMIC SERIES.

To expand $\log_a(1+x)$ in a series of ascending powers of x.

(1) Suppose $\log_a(1+x) = M + Ax + Bx^2 + Cx^3 + Dx^4 + \ldots.$ where M, A, B, C, D, &c. are at present unknown, and do not involve x.

Since the equation is true for every value of x, it is true when $x=0$, which gives:

$$\log_a 1 = M$$

$$\text{But} \quad \log_a 1 = 0 \qquad (\text{Art. } 4.)$$

$$\text{therefore } M = 0$$

$$\text{Hence } \log_a(1+x) = Ax + Bx^2 + Cx^3 + Dx^4 + \ldots..$$

(2) Again, the formula is true when x becomes $x+h$; that is, $\log_a(1+x+h) = A(x+h) + B(x+h)^2 + C(x+h)^3 + D(x+h)^4 \ldots\ldots$

and subtracting the preceding formula from this,

$$\log_a(1+x+h) - \log_a(1+x)$$
$$= Ah + B(2hx + h^2) + C(3x^2h + 3xh^2 + h^3)$$
$$+ D(4x^3h + 6x^2h^2 + 4xh^3 + h^4) + \ldots\ldots$$

$$\text{But } \log_a(1+x+h) - \log_a(1+x) = \log_a\left(\frac{1+x+h}{1+x}\right) \quad (\text{Art. } 6.)$$

$$= \log_a\left(1 + \frac{h}{1+x}\right)$$

Hence

$$\log_a\left(1+\frac{h}{1+x}\right) = Ah + B(2hx+h^2) + C(3x^2h+3xh^2+h^3) + D(4x^3h+6x^2h^2+4xh^3+h^4)+\ldots\ldots$$

(3) In the expression of Art. (1), let x become $\frac{h}{1+x}$, then $\log_a\left(1+\frac{h}{1+x}\right) = A\left(\frac{h}{1+x}\right) + B\left(\frac{h}{1+x}\right)^2 + C\left(\frac{h}{1+x}\right)^3 + D\left(\frac{h}{1+x}\right)^4 + \ldots\ldots$

(4) Equating together the expressions for $\log_a\left(1+\frac{h}{1+x}\right)$ in the last two articles,

$$Ah+B(2hx+h^2)+C(3x^2h+3xh^2+h^3)+D(4x^3h+6x^2h^2+4xh^3+h^4)+\ldots\ldots$$
$$=A\left(\frac{h}{1+x}\right)+B\left(\frac{h}{1+x}\right)^2+C\left(\frac{h}{1+x}\right)^3+D\left(\frac{h}{1+x}\right)^4+\ldots.$$

or, dividing by h,

$$A+B(2x+h)+C(3x^2+3xh+h^2)+D(4x^3+6x^2h+4xh^2+h^3)+\ldots\ldots$$
$$=A\left(\frac{1}{1+x}\right)+B\frac{h}{(1+x)^2}+C\frac{h^2}{(1+x)^3}+D\frac{h^3}{(1+x)^4}+\ldots\ldots$$

(5) Now this last equation must be true for all values of h. Let then $h=0$, and we have:

$$A+2Bx+3Cx^2+4Dx^3+\ldots. = \frac{A}{1+x}$$
$$=A(1-x+x^2-x^3+x^4-\ldots.)$$

(6) In this expression, equating like powers of x,

$$2B=-A,\ 3C=+A,\ 4D=-A,\ 5E=+A,\ \&c.$$

$$or\ B=-\frac{A}{2},\ C=+\frac{A}{3},\ D=-\frac{A}{4},\ E=+\frac{A}{5},\ \&c.$$

hence, $\log_a(1+x) = Ax + Bx^2 + Cx^3 + Dx^4 + Ex^5 - \ldots\ldots$

$$= A\left\{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots\ldots\right\}$$

(7) The value of the factor A can easily be determined by making $x = a-1$, or $1+x = a$

then $\log_a a$ or $1 = A\left\{(a-1) - \frac{(a-1)^2}{2} + \frac{(a-1)^3}{3} - \frac{(a-1)^4}{4} + \ldots\right\}$

or,

$$\frac{1}{A} = (a-1) - \tfrac{1}{2}(a-1)^2 + \tfrac{1}{3}(a-1)^3 - \tfrac{1}{4}(a-1)^4 + \tfrac{1}{5}(a-1)^5 - \ldots$$

(8) This expansion of $\log_a(1+x)$ is called the logarithmic series, from which, by giving successive values to x, the logarithms of all numbers to the base a might be determined.

In its present form, however, it is inconvenient, on account of the value of A, which would enter as a factor into every logarithm.

(9) To avoid this inconvenience let e be such a particular value of a, that the corresponding value of A may be equal to 1.

Then $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots$

(10) Therefore $\log_a(1+x) = A\log_e(1+x)$

$$A = \frac{\log_a(1+x)}{\log_e(1+x)}$$

$$\left.\begin{aligned} &= \log_a e \\ &= \frac{1}{\log_e a} \end{aligned}\right\} \text{ (by Art. 11.)}$$

(11) We have therefore arrived at this result:

$$\log_a(1+x) = A\left\{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\right\}$$

$$= \frac{1}{\log_e a}\left\{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\right\}$$

and $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots$

where a is any quantity, and e a certain value of it, that will be determined in the next Appendix.

(12) Cor. 1. $\text{Log}_e a = \log_e\{1+\overline{a-1}\}$
$$= (a-1) - \tfrac{1}{2}(a-1)^2 + \tfrac{1}{3}(a-1)^3 - \tfrac{1}{4}(a-1)^4 + \&c.$$
$$= \frac{1}{A} \text{ by Art. (7.)}$$

as at Art. (10.)

(13) Cor. 2. $\text{Log}_a(n+\delta) - \log_a n = \log_a\left(1+\frac{\delta}{n}\right)$
$$= A\left\{\frac{\delta}{n} - \tfrac{1}{2}\left(\frac{\delta}{n}\right)^2 + \tfrac{1}{3}\left(\frac{\delta}{n}\right)^3 - \ldots\right\}$$
$$= A\frac{\delta}{n} \text{ nearly,}$$

as $\frac{\delta}{n}$ is a very small fraction.

Similarly, $\log_a(n+\delta') - \log_a n = A\frac{\delta'}{n}$

Hence, $\log_a(n+\delta) - \log_a n : \log_a(n+\delta') - \log_a n = \delta : \delta'$.

This proportion, which is only approximately true, is used in constructing tables of proportional parts; as at Art. 24.

II.

THE EXPONENTIAL SERIES.

To expand a^x in a series of ascending powers of x.

(14)
$$\begin{aligned}
a^x &= \{1+(a-1)\}^x \\
&= 1+\frac{x}{1}(a-1)+\frac{x(x-1)}{1.2}(a-1)^2 \\
&\quad +\frac{x(x-1)(x-2)}{1.2.3}(a-1)^3+\ldots\ldots \\
&= 1+\{(a-1)-\tfrac{1}{2}(a-1)^2+\tfrac{1}{3}(a-1)^3-\tfrac{1}{4}(a-1)^4 \\
&\quad +\ldots.\}x \\
&\quad +\text{terms in } x^2,\ x^3,\ \ldots\ldots \\
&= 1+p_1x+p_2x^2+p_3x^3+p_4x^4+\&\text{c., suppose,}
\end{aligned}$$

where $p_1=(a-1)-\frac{1}{2}(a-1)^2+\frac{1}{3}(a-1)^3-\ldots\ldots$ and p_2, p_3, ... are yet to be determined.

(15) Since $a^x = 1+p_1x+p_2x^2+p_3x^3+p_4x^4+\ldots.$
therefore $a^z = 1+p_1z+p_2z^2+p_3z^3+p_4z^4+\ldots.$

Multiplying these two expressions together:

$$\begin{aligned}
a^{x+z}=1&+p_1(x+z) \\
&+p_1^2xz+p_2(x^2+z^2) \\
&+p_1p_2(x^2z+xz^2)+p_3(x^3+z^3) \\
&+p_1p_3(x^3z+xz^3)+p_2^2x^2z^2+p_4(x^4+z^4) \\
&+\&\text{c.}
\end{aligned}$$

(16) But
$$\begin{aligned}
a^{x+z} &= 1+p_1(x+z)+p_2(x+z)^2+p_3(x+z)^3 \\
&\quad +p_4(x+z)^4+\ldots\ldots \\
&= 1+p_1(x+z) \\
&\quad +p_2(x^2+2xz+z^2) \\
&\quad +p_3(x^3+3x^2z+3xz^2+z^3) \\
&\quad +p_4(x^4+4x^3z+6x^2z^2+4xz^3+z^4) \\
&\quad +\&\text{c.}
\end{aligned}$$

(17) The two expansions of a^{x+z}, given above, must be identical. Therefore equating like terms,

$$p_1^2 = 2p_2$$
$$p_1p_2 = 3p_3$$
$$p_1p_3 = 4p_4, \text{ and so on.}$$

or,
$$p_2 = \tfrac{1}{2}(p_1)^2$$
$$p_3 = \tfrac{1}{3}(p_1p_2) = \frac{1}{2.3}(p_1)^3$$
$$p_4 = \tfrac{1}{4}p_1p_3 = \frac{1}{2.3.4}(p_1)^4,$$

and so on.

But $p_1 = (a-1)-\frac{1}{2}(a-1)^2+\frac{1}{3}(a-1)^3-\&c.$
$= \log_e a$

Therefore $p_2 = \frac{1}{2}(\log_e a)^2$, $p_3 = \dfrac{1}{2.3}(\log_e a)^3$

$$p_4 = \frac{1}{2.3.4}(\log_e a)^4, \&c.$$

and $a^x = 1 + (\log_e a)\,x + (\log_e a)^2\dfrac{x^2}{1.2} + (\log_e a)^3\dfrac{x^3}{1.2.3}$
$+(\log_e a)^4\dfrac{x^4}{1.2.3.4} + \&c.$, which is the exponential series.

(18) Cor. 1. By making $a=e$, we have

$$e^x = 1+x+\frac{x^2}{1.2}+\frac{x^3}{1.2.3}+\frac{x^4}{1.2.3.4}+\ \ldots.$$

(19) Cor. 2. We can now determine the value of e. In the last expression, put $x=1$, then

$$e = 1+1+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\ \ldots..$$

and from this may be obtained

$$
\begin{aligned}
1+1 &= & &= 2\cdot \\
\frac{1}{1.2} &= \frac{1}{2} & &= \cdot 5 \\
\frac{1}{1.2.3} &= \frac{\cdot 5}{3} & &= \cdot 16666666 \\
\frac{1}{1.2.3.4} &= \frac{\cdot 16666666}{4} & &= \cdot 04166666 \\
\frac{1}{1.2.3.4.5} &= \frac{\cdot 04166666}{5} & &= \cdot 00833333 \\
\frac{1}{1.2.3.4.5.6} &= \frac{\cdot 00833333}{6} & &= \cdot 00138888 \\
\frac{1}{1.2.3....7} &= \frac{\cdot 00138888}{7} & &= \cdot 00019841 \\
\frac{1}{1.2.3....8} &= \frac{\cdot 00019841}{8} & &= \cdot 00002480 \\
\frac{1}{1.2.3....9} &= \frac{\cdot 00002480}{9} & &= \cdot 00000275 \\
\frac{1}{1.2.3....10} &= \frac{\cdot 00000275}{10} & &= \underline{\cdot 00000027,\ \&c.} \\
& & & \quad 2\cdot 71828176
\end{aligned}
$$

or, as far as the seventh place of decimals,

$$e = 2\cdot 7182818.$$

(20) This expression is the base of the natural system of logarithms, and it is adopted because it makes the logarithmic series and the exponential series much simpler than any other number would do.

Logarithms constructed to this base are called Napierian, from the discoverer, Napier of Merchistoun, who lived in the reign of James I.

They are also sometimes called Hyperbolic logarithms.

III.

THE COMPUTATION OF LOGARITHMS.

(21) We might obtain the Napierian logarithms from the series, $\log_e(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\&c.$ But this is not sufficiently convergent to be very serviceable.

If for x, we write $-x$, we shall have

$$\log_e(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}+\frac{x^4}{4}+\ \&c.$$

$$\text{or } \log_e\frac{1+x}{1-x}=\log_e(1+x)-\log_e(1-x)$$

$$=2\left\{x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+\ \cdots\cdots\right\}$$

$$\text{Let } \frac{1+x}{1-x}=\frac{y}{z}$$

$$\text{or } x=\frac{y-z}{y+z}$$

$$\text{Then } \log_e y=\log_e z+2\left\{\left(\frac{y-z}{y+z}\right)+\frac{1}{3}\left(\frac{y-z}{y+z}\right)^3+\frac{1}{5}\left(\frac{y-z}{y+z}\right)^5+\cdots\right\}$$

(22) Let $y=2$, $z=1$, or $\log_e z=0$.

$$\textit{Hence } \log_e 2=2\left\{\frac{1}{3}+\frac{1}{3}\left(\frac{1}{3}\right)^3+\frac{1}{5}\left(\frac{1}{3}\right)^5+\frac{1}{7}\left(\frac{1}{3}\right)^7+\ \cdots\cdot\right\}$$

$$\frac{1}{3} \qquad\qquad = \cdot 33333333$$

$$\frac{1}{3}\left(\frac{1}{3}\right)^3 = \frac{\cdot 33333333}{27} = \frac{\cdot 03703703}{3} = \cdot 01234567$$

$$\frac{1}{5}\left(\frac{1}{3}\right)^5 = \frac{\cdot 03703703}{9\times 5} = \frac{\cdot 00411522}{5} = \cdot 00082304$$

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$$\frac{1}{13}\left(\frac{1}{3}\right)^{13} = \frac{\cdot 00000564}{9\times 13} = \frac{\cdot 00000062}{13} = \cdot 00000005$$

&c.

$\cdot 34657356$

2

Therefore $\log_e 2 = \cdot 693147$

which is correct to the sixth place of decimals.

(23) In the above formula, put $y = 3$, $z = 2$.

$$\log_e 3 = \log_e 2 + 2\left\{\left(\frac{1}{5}\right) + \frac{1}{3}\left(\frac{1}{5}\right)^3 + \frac{1}{5}\left(\frac{1}{5}\right)^5 + \frac{1}{7}\left(\frac{1}{5}\right)^7 + \ldots\right\}$$
$$= \cdot 693147 + \cdot 405465$$
$$= 1\cdot 098612$$

(24) $\text{Log}_e 4 = 2\times\log_e 2$
$= 1\cdot 386294.$

(25) In the formula of Art. (2), let $y = 5$, $z = 4$

$$\log_e 5 = \log_e 4 + 2\left\{\frac{1}{9} + \frac{1}{3}\left(\frac{1}{9}\right)^3 + \frac{1}{5}\left(\frac{1}{9}\right)^5 + \ldots\ldots\right\}$$
$$= 1\cdot 386294 + \cdot 223144$$
$$= 1\cdot 609438.$$

(26) $$\begin{aligned}\text{Log}_e 6 &= \log_e (3\times 2) = \log_e 3 + \log_e 2\\ &= 1{\cdot}098612 + {\cdot}693147\\ &= 1{\cdot}791759.\end{aligned}$$

(27) $$\begin{aligned}\text{Log}_e 7 &= \log_e 6 + 2\left\{\frac{1}{13} + \frac{1}{3}\left(\frac{1}{13}\right)^3 + \frac{1}{5}\left(\frac{1}{13}\right)^5 + \dots\right\}\\ &= 1{\cdot}791759 + {\cdot}154151\\ &= 1{\cdot}945910.\end{aligned}$$

These are left for the student to complete. It is then evident that we may thus obtain the logarithms of all numbers to the base e.

(28) The Napierian logarithms being thus determined, we may find the logarithms of all numbers to any other base a by the formula:

$$\log_a y = \frac{1}{\log_e a} \log_e y,$$ Art. **10** and App. I. (11), the quantity $\frac{1}{\log_e a}$ being the modulus of the system of base a.

(29) In practice, only logarithms to the base 10 are used, for reasons stated at Art. **13** (called Briggs's, or common logarithms).

$$\begin{aligned}\text{Log}_e 10 &= \log_e 2 + \log_e 5\\ &= {\cdot}693147 + 1{\cdot}609438\\ &= 2{\cdot}302585.\end{aligned}$$

Therefore μ, the modulus of the common system,

$$\begin{aligned}&= \frac{1}{2{\cdot}302585}\\ &= {\cdot}434294.\end{aligned}$$

$$\begin{aligned}\text{And } \log_{10} z &= \mu \log_e z\\ &= {\cdot}434294 \log_e z.\end{aligned}$$

(30) Hence we may proceed to find the common logarithms of all numbers: the primes by the last formula; the others by the principle of Arts. 5, 6, 7, and 8.

$$\begin{aligned}
\text{Log}_{10}\, 2 &= \mu \log_e 2 = \cdot 434294 \times \cdot 693147 &&= \cdot 301030. \\
\text{Log}_{10}\, 3 &= \mu \log_e 3 = \cdot 434294 \times 1\cdot 098612 &&= \cdot 477121. \\
\text{Log}_{10}\, 4 &= 2 \log_{10} 2 = 2 \times \cdot 301030 &&= \cdot 602060. \\
\text{Log}_{10}\, 5 &= \log_{10} 10 - \log_{10} 2 = 1 - \cdot 301030 &&= \cdot 698970. \\
\text{Log}_{10}\, 6 &= \log_{10} 2 + \log_{10} 3 = \cdot 301030 + \cdot 477121 &&= \cdot 778151. \\
\text{Log}_{10}\, 7 &= \mu \log_e 7 = \cdot 434294 \times 1\cdot 945910 &&= \cdot 845098. \\
\text{Log}_{10}\, 8 &= \log_{10} 2 + \log_{10} 4 = \cdot 301030 + \cdot 602060 &&= \cdot 903090. \\
\text{Log}_{10}\, 9 &= 2 \log_{10} 3 = 2 \times \cdot 477121 &&= \cdot 954242.
\end{aligned}$$

And so on.

(31) Only the decimal part of these logarithms is tabulated; and they will answer for all natural numbers which differ only in the position of the point. (Art. **13.**) The characteristic is always discoverable by the principles of Arts. **18** and **20.**

And tables of proportional parts are made according to Arts. **24, 25**; and App. I. (13).

ANSWERS.

I.

(1) 625.

(2) 2 : 1.

(3) 27.

(4) $5\cdot\dot{3}$.

(5) -4.

(6) 1·5.

(7) ·778151, 2·607454, 3·653212, & 4·510544.

(8) 1·079181, 1·130334.

(9) 1·397940, 4·795880.

(10) ·477121, ·255273.

(11) 1·243038.

(12) $n = \dfrac{\log 2}{\log 3}$ to any base.

(13) $\dfrac{a \log 3}{\log 5}$ to any base.

(14) $\left.\begin{array}{l} \text{Log}\, x = \dfrac{\log c \times \log a^m}{\log (a^m b)} \\ \text{Log}\, y = \dfrac{\log c \times \log b}{\log (a^m b)} \end{array}\right\}$ to any base.

(15) $n = \dfrac{\log \{a + s(r-1)\} - \log a}{\log r}$

(16) $x = \dfrac{\log b}{m \log a}$

(17) $x = \left(\dfrac{m}{n}\right)^{\frac{n}{m-n}},\ y = \left(\dfrac{m}{n}\right)^{\frac{m}{m-n}}$

(18) $n = \sqrt{\dfrac{\log p}{\log a}}$

II.

(1) 4·871106.

(2) 1·409087.

(3) ·383995.

III.

(1) 4·8711057.

(2) 1·4090874.

(3) ·3839948.

(4) $\bar{1}$·940168.
(5) $\bar{3}$·403292.
(6) $\bar{6}$·322219.
(7) 418·1.
(8) 579500.
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(10) 2·260906.
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(13) $\bar{1}$·272331.
(14) $\bar{3}$·749230.
(15) 19·21971.
(16) ·03731086.
(17) ·00001654664.

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The answers of II. and III. have been thus arranged in corresponding lines, in order that the results obtained from tables of 6 figures and from those of 7 may be compared together. It will be seen that they do not altogether agree; thus bearing out the remark at Art. **26**, respecting the merely approximative truth of the proportional parts.

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(17) ·0257766.
(18) ·0000000036015.
(19) ·0085161.
(20) ·716807.
(21) ·159705.
(22) ·00000000000000014392.
(23) ·5709.
(24) $x = \cdot 95255$.
(25) ·28242.
(26) 44746·4
(27) ·395176.

VI.

(1) $x = 3\cdot 1273$.

(2) $x = -2\cdot 41575$.

(3) $x = \dfrac{\log (a^2 - b^2)}{\log\left\{\dfrac{(a+b)^2}{a-b}\right\}}$

(4) $x = 4{\cdot}17727$.

(5) $x = \dfrac{\log\{2m \pm \sqrt{4m^2 - 1}\}}{\log c}$.

(6) $x = 4{\cdot}2818$, $y = 3{\cdot}0584$.

(7) $x = \frac{3}{2}$, $y = \frac{5}{3}$.

(8) $x = 4{\cdot}718$, $y = 1{\cdot}887$.

(9) $x = 3{\cdot}5456$, $y = 1{\cdot}4182$.

(10) $x = 17{\cdot}0386$, $y = 2{\cdot}7837$.

(11) $x = \frac{9}{4}$, $y = \frac{27}{8}$.

(12) $x = 4{\cdot}335$, or $-{\cdot}335$.

(13) $n = 5$.

(14) 8 times.

(15) $n = 5$.

(16) $n = 7$.

(17) $n = 7$.

(18) $n = 6$.

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